SHU(MRU) 物理学院-每日一题 18

Prof. Shu

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题目 18.

推导一般性的洛伦兹变换 (K$'$ 系相对于 K 系的速度不平行于 x 轴). 设 $t' = t = 0$ 时,K$'$ 系和 K 系重合.

题目 17 的参考答案.

透镜的有效焦距满足

$$f = -\frac{\mathit{\Delta}}{f_1 f_2} = \frac{d - f_1 - f_2}{f_1 f_2} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{1}$$

其中

$$f_1 = (n_1 - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \quad f_2 = (n_2 - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

r_1, r_2 分别为两透镜的两个球面的曲率半径.

在波长改变 $\delta\lambda$ 时, 有

$$\begin{aligned} -\frac{\delta f_1}{f_1^2} &= \left(\frac{1}{r_1} - \frac{1}{r_2}\right)\frac{\mathrm{d}n_1}{\mathrm{d}\lambda}\delta\lambda = \frac{D_1}{(n_1 - 1)f_1}\delta\lambda \\ -\frac{\delta f_2}{f_2^2} &= \left(\frac{1}{r_1} - \frac{1}{r_2}\right)\frac{\mathrm{d}n_2}{\mathrm{d}\lambda}\delta\lambda = \frac{D_2}{(n_2 - 1)f_2}\delta\lambda \end{aligned} \tag{2}$$

对 (1) 求微分, 并代入 (2), 可得

$$\begin{aligned}-\frac{\delta f}{f^2} &= -\frac{\delta f_1}{f_1^2} - \frac{\delta f_2}{f_2^2} + \frac{d\delta f_1}{f_1^2 f_2} + \frac{d\delta f_1}{f_1^2 f_2} \\ &= -\left[\frac{D_1}{(n_1-1)f_1} + \frac{D_2}{(n_2-1)f_2}\right]\delta\lambda \\ &\quad + \left[\frac{D_1}{n_1-1} + \frac{D_2}{n_2-1}\right]\frac{d}{f_1 f_2}\delta\lambda\end{aligned}$$

当透镜组无色差时, 有 $\delta f = 0$, 即

$$\frac{D_1}{(n_1-1)f_1} + \frac{D_2}{(n_2-1)f_2} = \left[\frac{D_1}{n_1-1} + \frac{D_2}{n_2-1}\right]\frac{d}{f_1 f_2}$$

解得

$$d = \frac{\frac{f_2 D_1}{n_1-1} + \frac{f_1 D_2}{n_2-1}}{\frac{D_1}{n_1-1} + \frac{D_2}{n_2-1}} \tag{3}$$

特别地, 当 $n_1 = n_2$, $D_1 = D_2$ 时, $d = (f_1 + f_2)/2$.