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题目：设顺磁介质遵守居里定律$M = AVH/T$，无外场时的比热容是$C_0 = b/T^2$，试求：

（1）C_H和C_M的表达式；

（2）证明内能只是温度的函数；

（3）绝热磁化率η的表达式。

C_H为磁场不变时的热容量，C_M为磁化强度不变时的热容量，A，b 均为常数。

解答：（1）为了防止磁场与焓混淆，下面磁场将小写。下面取（M,T）坐标。首先说明C_M仅仅是 T 的函数。由热力学基本方程：

$$dU(M,T) = TdS + \mu_0 h dM = T\frac{\partial S(M,T)}{\partial T}dT + [T\frac{\partial S(M,T)}{\partial M} + \mu_0 h(M,T)]dM$$

$$\Rightarrow \frac{\partial}{\partial M}[T\frac{\partial S(M,T)}{\partial T}] = \frac{\partial}{\partial T}[T\frac{\partial S(M,T)}{\partial M} + \mu_0 h(M,T)]$$

$$\Rightarrow \frac{\partial S(M,T)}{\partial M} + \mu_0\frac{\partial h(M,T)}{\partial T} = 0$$

$$\frac{\partial C_M(M,T)}{\partial M} = \frac{\partial}{\partial M}[T\frac{\partial S(M,T)}{\partial T}] = T\frac{\partial}{\partial T}\frac{\partial S(M,T)}{\partial M} = T\frac{\partial^2}{\partial T^2}h(M,T) = 0$$

其次说明在无外场的时候，C_H和C_M在数值上相等。

$$C_h(M,T) - C_M(T) = \mu_0 T[\frac{\partial H(M,T)}{\partial T}]^2\frac{\partial M(h,T)}{\partial h} = \frac{\mu_0 M^2}{AV}$$

以上推导利用汪志诚《热力学与统计物理》2.19 题结论，可见当无外场即 M=H=0 时两者数

值相等。因此：

$$C_M(T) = C_0 V = \frac{bV}{T^2}$$

$$C_h(M,T) = C_M(T) + \frac{\mu_0 M^2}{AV} = \frac{bV}{T^2} + \frac{\mu_0 M^2}{AV}$$

（2）利用上一问结论：

$$\frac{\partial U(M,T)}{\partial M} = T\frac{\partial S(M,T)}{\partial M} + \mu_0 h(M,T) = -\mu_0\frac{\partial h(M,T)}{\partial T} + \mu_0 h(M,T) = 0$$

（3）

$$\eta = \frac{\partial(S,M)}{\partial(S,h)} = \frac{\partial(S,M)}{\partial(T,M)}\frac{\partial(T,M)}{\partial(T,h)}\frac{\partial(T,h)}{\partial(S,h)} = \frac{C_M}{C_h}\frac{\partial M(h,T)}{\partial h} = \frac{AV}{T}(1 + \frac{\mu_0 M^2 T^2}{AbV})^{-1}$$